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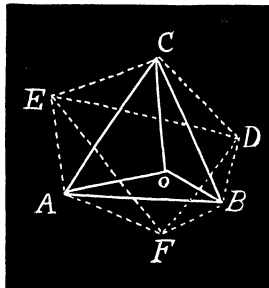
sides and included angle of OAB we find

$$AB = \sqrt{31^2 + 28^2 - 2 \times 31 \times 28 \cos(60^\circ + 39^\circ 12')} = 44.97 +, \text{ the side required.}$$

IV. Solution by J. W. WATSON, Middle Creek, Ohio.

Let ABC be the given triangle, o the given point. Put $Ao = x = 28$, $Bo = b = 20$ and $Co = c = 31$. Let $S =$ a side of the triangle. Then $\text{area} = \frac{1}{2} S^2 \sqrt{3}$.

Draw the $\triangle AFB = \triangle AoB$, $\triangle BDC = \triangle BoC$ and $\triangle AEC = \triangle AoC$. With A as a center and a as a radius describe the arc EoK . With B as a center and b as a radius describe arc FOD . With C as a center and c as a radius, describe arc EoD . Join EF , FD and DE . Now $\angle EAF = 2\angle CAB = 120^\circ$. Therefore, EF is equal to a side of an equilateral triangle inscribed in a circle whose radius is a .



$\therefore EF = a\sqrt{3}$. In same manner find $FD = b\sqrt{3}$ and $ED = c\sqrt{3}$. Area $\triangle EAF = 196\sqrt{3}$, of $\triangle FBD = 100\sqrt{3}$, of $\triangle DCE = 240\frac{1}{2}\sqrt{3}$, of $\triangle EFD = 823.18129 +$. The area of the entire polygon $AFBDC E = 1751.99357 + \text{sq. rds.}$ This is double the area of $\triangle ABC$.

$\therefore \text{area } \triangle ABC = 875.996785 \text{ sq. rds.}$ Equating the two expressions for the area, we have $\frac{1}{2} S^2 \sqrt{3} = 875.996785$. $\therefore S = 44.97 + \text{rods, side of the triangle.}$

23. Proposed by ROBERT J. ALEY, A. M., Professor of Mathematics, Indiana University, Bloomington, Indiana.

Sum to n terms the following series: $11 + 25 + 45 + 71 + 103 + \dots$

I. Solution by B. F. BURLERSON, Oneida Castle, New York, D. G. DORRANCE, Jr., Camden, New York, J. H. GROVE, Professor of Mathematics, in Howard College, Brownwood, Texas, and J. A. TIMMONS, A. M., Professor of Mathematics in St. Mary's College, St. Mary's, Kentucky.

The first differences are $14, 20, 26, 32, \dots$;

the second differences are $6, 6, 6, \dots$;

the third differences are $0, 0, \dots$

Putting d_1, d_2, d_3 for the first terms of the differences we have

$$d_1 = 14, d_2 = 6, d_3 = 0.$$

Then, by the "Differential Method," we have, if we put a for the first term of the series, and S for its sum,

$$\begin{aligned} S &= na + \frac{1}{2}n(n-1)d_1 + \frac{1}{6}n(n-1)(n-2)d_2, \\ &= 11n + 7n(n-1) + n(n-1)(n-2), \\ &= n^3 + 4n^2 + 6n = n(n^2 + 4n + 6) = n[(n+2)^2 + 2]. \end{aligned}$$

Similarly solved by P. S. BERG, J. A. CALDERHEAD, H. W. DRAUGHON, J. H. DRUMMOND, J. K. ELLWOOD, A. L. FOOTE, M. A. GRUBER, ARTEMAS MARTIN, F. P. MATZ, COOPER D. SCHMITT, H. C. WHITAKER, and G. B. M. ZERR.

II. Solution by J. F. W. SCHEFFER, Hagerstown, Maryland.

This is an arithmetical progression of the 2d order, in which $a = 11$,

$\triangle a = 14$, $\triangle^2 a = 6$, $\triangle^3 a = 0$. $\therefore S = 11n + \left(\frac{n}{2}\right) 14 + \left(\frac{n}{3}\right) 6 = n(n^2 + 4n + 6)$, or $n[(n+2)^2 + 2]$.

III Solution by A. C. ROBERTS, Long Bottom, Ohio; C. E. WHITE, Trafalgar, Indiana; and OTTO GECKELER, Bloomington, Indiana.

Here $S = n[(n+2)^2 + 2]$, and this formula is derived as follows:

The series $11 + 25 + 45 + 71 + 103 + 141 + 185 + \dots$ consists of the sum of two series as follows:

1st $11[1 + 2 + 3 + 4 + \dots + n]$ whose sum $= 11\left(\frac{n(n+1)}{2}\right)$

2d $3[1 + 4 + 3^2 + 16 + 5^2 + \dots + (n-1)^2]$ whose sum $= \frac{1}{2}[n(n-1)(2n-1)]$.

$\therefore$ the entire sum $= \frac{1}{2}[n(n+1)] + \frac{1}{2}[n(n-1)(2n-1)] = n^3 + 4n^2 + 6n$.

$$n^3 + 4n^2 + 6n = n[n^2 + 4n + 6] = n[(n+2)^2 + 2].$$

IV. Solution by OTTO CLAYTON, A. B., Indiana University, Bloomington, Indiana, and JOHN, B. FAUGHT, Bloomington, Indiana.

Making a table as follows:

$$11 + \left\{ \begin{array}{c} 11 \\ 14 \end{array} \right\} + \left\{ \begin{array}{c} 11 \\ 14 \\ 14+6 \end{array} \right\} + \left\{ \begin{array}{c} 11 \\ 14 \\ 14+6 \\ 14+6+6 \end{array} \right\} + \left\{ \begin{array}{c} 11 \\ 14 \\ 14+6 \\ 14+6+6 \\ 14+6+6+6 \end{array} \right\},$$

we see that the n th or general term is $11 + (n-1).14 + \frac{(n-1)(n-2)}{2}.6$

$$S_n = 11n + \sum_{i=1}^{n-1} (n-1).14 + \sum_{i=1}^{n-1} \frac{(n-1)(n-2)}{2}.6 = 11n + \frac{n(n-1)}{2}.14 + \left\{ n-2 + \frac{(n-2)(n-3)}{2}.2 + \frac{(n-2)(n-3)(n-4)}{6}.6 \right\}.6 = n(n^2 + 4n + 6), \text{ or, } n[(n+2)^2 + 2].$$

V. Solution by W. WIGGINS, Richmond, Indiana, and the PROPOSER.

By inspection, the general term $U^n = 3n^2 + 5n + 3$.

Now take another series whose general term $V_n = n^3 + (n-1)^2 + 3n$.

$$\triangle V_n = [(n+1)^3 + n^2 + 3(n+1)] - [n^3 + (n-1)^2 + 3n] = 3n^2 + 5n + 3 = U_n.$$

By Chrystal, Chapter xxxi, Sec. 3, $\sum_{n=1}^n U_{n+1} = V_n - V_0$, $\sum_{n=1}^n U_n = V_n - V_1$, $\therefore \sum_{n=1}^n U_n = V_n - 1 + V_1 = (n+1)^3 + n^2 + 3(n+1) - 4 = n^3 + 4n^2 + 6n = n[(n+2)^2 + 2]$.

PROBLEMS.

34. Proposed by ROBERT J. ALEY, A. M., Professor of Mathematics, Indiana University, Bloomington, Indiana.

$$\sum_1^n \frac{(n+2)^2}{n(n+4)} = \text{what?}$$

35. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Prove that the product of two numbers, each the sum of (4) squares may be expressed as the sum of four squares in 48 different ways and unite some or all of the 48 ways.

Solutions to these problems should be received on or before December 1st.